Renormalizable Electrodynamics of Vector Mesons

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It is shown that the conventional theory of charged spin-one mesons interacting with photons can be renormalized provided meson mass m and charge e are restricted by the relation $Z(m^2,e^2)=0$, where Z is the meson wave-function renormalization constant.

1. INTRODUCTION

IT is the purpose of this paper to show that conventional theory of charged spin-one mesons interacting with photons (vector electrodynamics) can be renormalized provided the meson mass m and the physical coupling constant e are restricted by the relation

$$Z(m^2,e^2)=0, \tag{1}$$

where Z is the meson wave-function renormalization constant. The essence of the proof lies in showing that if (1) is satisfied, the modified vertex function Γ_1 behaves like $\sim 1/p$ and the modified propagator Δ_1 like ~ 1 for large p. Thus if S-matrix elements are computed as in Dyson's method, by first drawing irreducible diagrams and then writing Δ_1 and Γ_1 for each line and each vertex, the resulting integrals are all finite (except possibly those for meson self-mass and photon wave-function renormalization constant).

In Secs. 2 and 3 the necessary formalism is developed; Sec. 4 outlines the proof and Sec. 5 is concerned with the implications of relation (1). In a separate paper with R. Delbourgo we give actual computations of Δ_1 and Γ_1.

2. THE PROPAGATOR

Let the (renormalized) fields $A_\mu^\pm$ describe charged stable vector particles of mass m. The conventional Lagrangian for vector electrodynamics is

$$L=-\tfrac{1}{2}ZF_{\mu\nu}{}^+F_{\mu\nu}{}^- - Zm_0{}^2A_\mu{}^+A_\mu{}^- - \tfrac{1}{4}F_{\mu\nu}{}^0F_{\mu\nu}{}^0, \tag{2}$$

where

$$F_{\mu\nu}{}^\pm=\partial_\mu{}^\pm A_\nu{}^\pm-\partial_\nu{}^\pm A_\mu{}^\pm,$$
$$\partial_\mu{}^\pm=\partial/\partial x_\mu \mp ieA_\mu{}^0.$$

$A_\mu{}^0$ is the photon field and Z and $m_0{}^2$ are constants specified below.

We write the Fourier transform of the propagator[1] $\langle A_\mu{}^+(x)A_\nu{}^-(y)\rangle_+$ in the form

$$\Delta_{1\mu\nu}(p)=d_{\mu\nu}\lambda_1(p^2)+e_{\mu\nu}\lambda_2(p^2), \tag{3}$$

where[2]

$$d_{\mu\nu}=(-\delta_{\mu\nu}+p_\mu p_\nu/p^2), \tag{4}$$

$$e_{\mu\nu}=p_\mu p_\nu/p^2. \tag{5}$$

By hypothesis the spin-one part of Δ_1 has a pole at $p^2=m^2$ with "residue" $d_{\mu\nu}$. Thus $\lambda_1(p^2)$ must have the form

$$\lambda_1{}^{-1}(p^2)=(p^2-m^2)Z(p^2), \tag{6}$$

where $Z(p^2)$ equals

$$Z(p^2)=1-(p^2-m^2)\int\frac{G_1(K^2)dK^2}{p^2-K^2+i\epsilon}. \tag{7}$$

Also the condition that there is no pole at $p^2=0$, means

$$\lambda_1(0)+\lambda_2(0)=0. \tag{8}$$

One may, therefore, write[3]:

$$\lambda_2{}^{-1}(p^2)=-\lambda_1{}^{-1}(0)-m^2p^2\int\frac{G_2(K^2)dK^2}{p^2-K^2+i\epsilon}. \tag{9}$$

Note that with (6) and (9)

$$\lim_{p^2\to m^2}(p^2-m^2)\Delta_{1\mu\nu}=(-\delta_{\mu\nu}+p_\mu p_\nu/m^2).$$

We now define the constants Z and $m_0{}^2$ which occur in the Lagrangian. Let[4]

$$Z=\lim_{p^2\to\infty}Z(p^2)=1-\int G_1dK^2, \tag{10}$$

$$Zm_0{}^2=\lim_{p^2\to\infty}\lambda_2{}^{-1}(p^2)=m^2\left(1-\int\frac{m^2}{K^2}G_1-\int G_2\right). \tag{11}$$

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[1] We follow the notation of the excellent paper by K. W. Ford, Nuovo Cimento 24, 1671 (1962).

[2] Writing $\mathbf{d}$ and $\mathbf{e}$ for $d_{\mu\nu}$ and $e_{\mu\nu}$, note that $-\mathbf{e}+\mathbf{d}=-1$, $\mathbf{dd}=-\mathbf{d}$, $\mathbf{ee}=\mathbf{e}$, $\mathbf{de}=\mathbf{ed}=0$. Also if $\mathbf{\Delta}=\lambda_1\mathbf{d}+\lambda_2\mathbf{e}$, then $\mathbf{\Delta}^{-1}=\lambda_1{}^{-1}\mathbf{d}+\lambda_2{}^{-1}\mathbf{e}$.

[3] For theories where conservation laws of the type $\partial J^\pm/\partial x_\mu=0$ hold, $G_2=0$ and λ_2 is a constant. This clearly is not the case for the present theory.

[4] Canonical commutation relations give alternative (but equivalent) expressions for Z and $Zm_0{}^2$. Thus comparing the canonical values of $[A_k(\mathbf{x}),A_l(\mathbf{x}')]$ and $[A_k(\mathbf{x}),\dot{A}_l{}^-(\mathbf{x}')]$, $(k,l=1, 2, 3)$ with those deduced from dis. $\Delta_1(x-x')$ one obtains:

$$Z^{-1}=1+\int G_1(K^2)|Z(K^2)|^{-2}dK^2, \tag{A}$$

$$\frac{1}{Zm_0{}^2}=\frac{1}{m^2}\int[G_1(K^2)|Z(K^2)|^{-2}+K^2G_2(K^2)|\lambda_2(K^2)|^2m^2]\frac{dK^2}{K^2}. \tag{B}$$

To establish equivalence of (10) with (A) for example, note that

$$\mathrm{Im}Z^{-1}(p)=|Z(p^2)|^{-2}G(p^2),$$

so that,

$$Z^{-1}(p^2)=1+(p^2-m^2)\int\frac{G_1|Z(K^2)|^2}{p^2-K^2+i\epsilon}dK^2,$$

provided the last integral converges. Comparison with (7) in the limit $p^2\to\infty$ proves the equivalence of (10) and (A).

The canonical expression for $[A_k{}^+(\mathbf{x}),\dot{A}_l{}^-(\mathbf{x}')]$, has been implicitly given by G. Wentzel, *Quantum Theory of Fields* (Interscience Publishers, Inc., New York), p. 93. To obtain it use the identity

$$\dot{A}_l{}^\pm=Z^{-1}[\pi_l{}^\mp-(1/m_0{}^2)\partial_l{}^\pm\partial_j{}^\pm\pi_j{}^\mp]-ieA_4{}^0A_l{}^\pm, \tag{C}$$

which can be derived from the equations of motion. The canonical momenta π_j occurring in (C) satisfy,

$$[A_k{}^\pm(\mathbf{x}),\pi_j{}^\pm(\mathbf{x}')]=i\delta_{ij}\delta(\mathbf{x}-\mathbf{x}').$$

Thus finally

$$\lambda_2^{-1}(p^2)=Zm_0^2-m^2\int\frac{K^2G_2(K^2)}{p^2-K^2+i\epsilon}. \qquad (12)$$

3. THE VERTEX FUNCTION

For the vertex function $\Gamma_{\mu\nu 1}{}^{a}(p,p')$, by considering the product $\Box_Z(\partial/\partial z_a)\langle A_\mu{}^+(x)A_\nu{}^-(y)A_a{}^0(z)\rangle_+$, one deduces the Ward-Takahashi identity

$$\Delta_1^{-1}(p)-\Delta_1^{-1}(p')=-(p-p')_a\Gamma_1{}^a(p,p') \qquad (13)$$

and its differential form

$$\partial\Delta_1^{-1}/\partial p_a=-\Gamma_1{}^a(p,p). \qquad (14)$$

Also from charge-conjugation invariance

$$\Gamma_{\mu\nu 1}(p,p')=-\Gamma_{\nu\mu 1}{}^a(-p',-p). \qquad (15)$$

Equation (13) can be solved to give

$$\Gamma_1=\Gamma_A+\Gamma_B.$$

Here Γ_B is an arbitrary function which satisfies[5] $(p-p')_a\Gamma_B{}^a=0$, and

$$-\Gamma_A=\frac{(p+p')_a}{p^2-p'^2}[\Delta^{-1}(p)-\Delta^{-1}(p')]. \qquad (16)$$

Explicitly,

$$\Delta^{-1}(p)-\Delta^{-1}(p')=(p^2-p'^2)A_1(p^2p'^2)+[p_\mu p_\nu X(p^2)-p_\mu'p_\nu'X(p'^2)], \qquad (17)$$

where

$$A_1(p^2,p'^2)=Z+\int\frac{(K^2-m^2)^2G_1}{(p^2-K^2)(p'^2-K^2)}dK^2,$$
$$X(p^2)=Z-\int\frac{(K^2-m^2)^2G_1}{K^2(p^2-K^2)}dK^2-m^2\int\frac{G_2dK^2}{p^2-K^2}. \qquad (18)$$

In general, all integrals involved in Δ and Γ_A converge provided[6]

$$\int G_1dK^2<\infty,\quad \int G_2dK^2<\infty. \qquad (19)$$

Now if e and m are so related that[7]

$$Z(e^2,m^2)=0,$$

and if we make (at this stage, *ad hoc*) assumption that

$$\lim_{K^2\to\infty}G_1\sim(1/K^2)^2, \qquad (20)$$

Eq. (18) shows that for large p or p', A_1, X, etc., have the form

$$A_1(p^2,p'^2)\sim 1/(ap^2+bp'^2),$$
$$X(p^2)\sim 1/p^2,$$

and therefore

$$\Gamma_1\sim 1/(\alpha p+\alpha' p'). \qquad (21)$$

The same conditions ensure that

$$Z(p^2)\sim 1/p^2 \quad\text{and}\quad \Delta_1\sim 1. \qquad (22)$$

In the Sec. 4, we consider the validity of (20).[8]

4. INTEGRAL EQUATIONS FOR Γ_1 AND Δ_1

To set up the coupled equations Δ_1 and Γ_1 and to write general scattering matrix elements, we follow Dyson's method and split off from L the conventional free Lagrangian L_0 which forms the basis of the interaction representation.[9] {Notice that the interaction Lagrangian contains (nondivergent) self-mass terms as well as kinetic energy terms of the type $(Z-1)A_\mu{}^+\times[(p^2-m^2)d_{\mu\nu}+m^2e_{\mu\nu}]A_\nu{}^-$.} Instead of writing S-matrix elements in terms of the free propagator

$$\Delta_{F0}=\mathbf{d}/(p^2-m^2)+(1/m^2)\mathbf{e}$$

and the unmodified vertex,[10]

$$\Gamma_0=\delta_{\mu\nu}(p+p')_a-\delta_{\mu a}p_\nu-\delta_{\nu a}p_\mu',$$

we first compute Δ_1 and Γ_1 as solutions of the integral equations below which are derived from the given Lagrangian and then write down other S-matrix elements by drawing irreducible graphs and by inserting in these Δ_1 and Γ_1 for the lines and the vertices.

The integral equations for Γ_1 and Δ_1 are

$$\Gamma_1(p,p')=Z\Gamma_0(p,p')+K(p,p'), \qquad (23)$$

where

$$K=e^2\int\Gamma_1(e)\Delta_1(e)\Gamma_1(e)\Delta_1(e)\Gamma_1(e)D_1(e)+e^4\int\cdots, \qquad (24)$$

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[5] Γ_B [which contains the dependence of Γ_1 on $(p-p')=t$] must have the general form:

$$\Gamma_B=[t_a(p^2-p'^2)-t^2(p+p')_a]\times[\delta_{\mu\nu}F_1+p_\mu p_\nu'F_2+p_\mu t_\nu F_3+p_\mu't_\nu F_3'+t_\mu t_\nu F_4]+(\delta_{a\mu}t_\nu-\delta_{a\nu}t_\mu)F_5+(t^2\delta_{a\mu}-t_at_\mu)p_\nu'F_6-(t^2\delta_{a\nu}-t_at_\nu)p_\mu F_6'+(t^2\delta_{a\mu}-t_at_\mu)t_\nu F_7-(t^2\delta_{a\nu}-t_at_\nu)t_\mu F_7',$$

where, using (15), the invariant functions F_1, F_2, F_4, F_5 are symmetric in p and p', and for F_3, F_6, and F_7, $F'(p^2,p'^2,t^2)=F(p'^2,p^2,t^2)$. F_5 gives the magnetic moment and F_4 the quadrupole moment of the vector particle. Further on we make the approximation $\Gamma\approx\Gamma_A$.

[6] This means both Z and Zm_0^2 are finite. Note that m_0^2 always occurs multiplied by the constant Z.

[7] With zero-photon mass there is no mass other than m in the present theory. Thus, the relation must reduce to $z(e^2)=0$.

[8] All these statements are accurate to the extent that powers of $(\ln p^2)$ are ignored.

[9] For details of the procedure see P. T. Matthews and A. Salam, Phys. Rev. **94**, 185 (1954). One would get the same Eqs. (23) and (25) if Schwinger's Green's function method is used with the Z-containing Lagrangian (2), [J. Schwinger, Proc. Natl. Acad. Sci. **37**, 452 (1951)].

[10] Throughout this paper we have consistently ignored the so-called "Compton parts," i.e., the modifications of the 2-meson 2-photon vertices which occur in electrodynamics of bose particles. Since Ward-Takahashi identities hold also for these graphs, their high-energy behavior presents no new conceptual difficulties. In this respect the β formalism for vector electrodynamics would have been superior to the formalism of this paper because no "Compton part" insertions are necessary in that case.

and[11,12]

$$\Delta_{\mu\nu 1}^{-1}(p) = -(p-p')_\alpha \times \int_0^1 \Gamma_{\mu\nu 1}^{\alpha}(px+p'(1-x),\, px+p'(1-x))dx \quad (25)$$

with $p'^2=m^2$, and all terms involving $p_\mu{}^1$, $p_\nu{}'$ omitted.

To solve (23) and (25), first consider the case $Z\neq 0$. The following approximation procedure reproduces the conventional perturbation series: (i) Take $Z\Gamma_0(p,p')$ as the first approximation to Γ_1. (ii) Integrate Eq. (25); the first approximation to Δ_1^{-1} therefore is $Z\Delta_{F0}^{-1}$. Since $\Delta_1\mathbf{d}$ has a pole at $p^2=m^2$ with residue $=\mathbf{d}$, to this order $Z=1$. (iii) Use Γ_0 and Δ_{F0} in (24) to obtain the next approximation[13] to Γ_1, Δ_1 (and Z), and so on.

Since $\Gamma_0\sim(p+p')$, it is clear that for the inhomogeneous case ($Z\neq 0$), Γ_1 is unlikely to converge faster than $(p+p')$.

If $Z=0$, we show below that the situation so far as the high-energy behavior is concerned is completely different. However, one may still set up an approximation scheme similar to the above, with only the change that in the zeroth approximation $Z\Delta_{F0}^{-1}$ is to be replaced by a suitable $(\Delta_1^{(0)})^{-1}$. Thus, in an obvious notation: (i) Take $\Gamma_1^{(0)}=\Gamma_A$ as defined in (17) with two unknown functions λ_1 and λ_2. Integrate (25) to get $\Delta_1^{(0)}=\lambda_1\mathbf{d}+\lambda_2\mathbf{e}$. (ii) Insert Γ_A for Γ_1 on the right hand side of (24) (fixing for practical purposes on some suitable subset of irreducible graphs). This gives:

$$\Gamma_1(p,p')=\Gamma_A(p,p')+\Gamma_B(p,p')=K[\Gamma_A]. \quad (26)$$

For $p=p'$, $\Gamma_B=0$. Thus,

$$\Gamma_A(p,p)=\frac{\partial}{\partial p}\Delta^{-1}(p)=K[\Gamma_A]_{p=p'}. \quad (27)$$

This is a set of homogeneous equations for λ_1 and λ_2 or, equivalently, G_1 and G_2. Once these are solved, (26) for $p\neq p'$ gives the zeroth approximation to Γ_B; the next approximations to G_1, G_2, and Γ_B are obtained by successive substitutions in $K[\Gamma_1]$.

The crucial step for the entire procedure then is the initial determination of $G_1^{(0)}$ and $G_2^{(0)}$ solutions of (27).

In a second paper (with R. Delbourgo) we present these solutions and show that $G_1^{(0)}$ and $G_2^{(0)}$ do indeed satisfy the convergence criteria of (19) and (20).[14] Here we show that if the initial approximations $G_1^{(0)}$, $G_2^{(0)}$ satisfy (19) and (20), all successive approximations possess the same property, and that $\Gamma_1(p,p')$ falls essentially as $1/p$ (or $1/p'$) when either of the variables p or p' is large. The proof is elementary. Since $\Delta_1^{(0)}(p)\sim 1$ by hypothesis, we infer from (17) that $\Gamma_A^{(0)}\sim 1/p$. Assuming that the photon propagator[12] $D_1(p^2)\sim 1/p^2$, one can see that the integral on the right of (26) must converge, yielding $\Gamma_1^{(1)}\approx 1/(\alpha p+p'\alpha')$ so far as dimensions are concerned. From (25) this means that the next approximation is $\Delta_1^{(1)}(p)\sim 1$,[15] and that $G_1^{(1)}$ and $G_2^{(1)}$ satisfy (19) and (20).

Before closing this section, we estimate the dimensional behavior of any Feynman integral with E_m external meson and E_p external photon lines. With $\Gamma\sim 1/p$ and $\Delta_1\sim 1$, these integrals converge provided[16,17]

$$2E_m+E_p>4.$$

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[11] Equation (25), which is the integral equivalent of (13), was first derived by J. C. Ward, Phys. Rev. **84**, 897 (1951). Instead one may work with the Dyson equation:

$$\Delta^{-1}=[Z(p^2-m_0^2)\mathbf{d}+Zm_0^2\mathbf{e}]-\pi_1^*(p), \quad (A)$$

where

$$\pi_1^*=\sum_{\substack{\text{sum over categories}\\ \text{of graphs}}} e^2\int \Gamma_0\Delta_1D_1\Gamma_0+\cdots. \quad (B)$$

"Categories" which here take the place of "irreducible" graphs have been defined by A. Salam, Phys. Rev. **82**, 217 (1951). Equation (A) is more general in so far as it applies also to nongauge-invariant theories. However, its disadvantage is the explicit appearance of Γ_0 on the right-hand side of (B). As is well known from the analysis of the "overlapping" self-energy parts π_1^* has the same behavior as $\sim\int\Gamma_1\Delta_1\Gamma_1D_1$.

[12] More precisely, one should also write an integral equation for the photon propagator $D_1(p)$ and solve it simultaneously with the equations for Γ_1 and Δ_1. For purposes of the present paper this is an unnecessary complication.

[13] To maintain gauge invariance and for consistency with (25), the set of irreducible graphs retained at each stage of approximation should include also graphs made up from appropriate "Compton-parts." These points will be covered in a second paper.

[14] The chief difficulty of solving (27) lies in the imposition of $Z(e^2,m^2)=0$. This is because Z itself is being computed (in terms of G_1) at the same time as the equation is being solved. It is worth noticing that it is only the real part of Δ^{-1} or Γ [see Eq. (23) or Eq. (A) of footnote 11] which explicitly depends on Z. Thus it is the high-energy behavior of only the real part which is in error unless we use $Z(e^2,m^2)=0$.

[15] This also means $Z^{(1)}(p^2)\sim 1/p^2$ so that

$$Z^{(1)}=\lim_{p^2\to\infty}Z(p^2)\equiv 0.$$

In other words, one does not improve on e^2 deduced from $Z^{(0)}(e^2,m^2)=0$ unless more irreducible graphs are included in the approximation to K.

[16] To see how this works out in practice, consider the simple case of a single closed loop with n external photon lines ($n>4$) of momenta $k_1,\cdots,k_n$. The Feynman integral has the form:

$$F(k_1,\cdots,k_n)=\int d^4p\,[\Gamma_1(p,\,p+k)]^n[\Delta_1(p+k)]^n.$$

For large p, the behavior of the integrand is dominated by the basic unit

$$\Gamma_1(p,p)\Delta_1(p)=\left(\frac{\partial}{\partial p}\Delta_1^{-1}(p)\right)\Delta_1(p)$$
$$=(\lambda_1'\mathbf{d}+\lambda_2'\mathbf{e}+\lambda_1\mathbf{d}'+\lambda_2\mathbf{e}')(\lambda_1^{-1}\mathbf{d}+\lambda_2^{-1}\mathbf{e})$$
$$\approx(1/p)+(1/p)(\lambda_2/\lambda_1+\lambda_1/\lambda_2).$$

For unmodified Δ_{F0} and Γ_0, $\lambda_1\sim 1/p^2$, $\lambda_2\sim 1$ so that the basic unit $\Gamma_0\Delta_{F0}$ has the behavior

$$\Gamma_0\Delta_{F0}\approx p.$$

If, however, (19)–(20) are satisfied, so that λ_1, $\lambda_2\sim 1$ then

$$\Gamma_1\Delta_1\sim 1/p$$

and the closed-loop integral $F(k_1,\cdots,k_n)$ ($n>4$) is convergent. The moral of the above analysis for the renormalization of electrodynamics of higher spin particles is clear. For a Yukawa-type theory, renormalizability needs an increase rate of $\Gamma_1\Delta_1$ no faster than $1/p$. A necessary condition for this seems to be that neither one of the two "orthogonal" functions λ_1 and λ_2 (of which Δ_1 is made up) should be more convergent than the other.

[17] For the conventional theory of scalar mesons interacting with photons and with no relation like $Z(e^2,m^2)=0$ operative, the

This means that the only possible infinite integrals are those corresponding to photon wave-function renormalization and meson self-mass[18] and all other S-matrix elements are finite.

5. THE RELATION $Z(e^2,m^2)=0$

It is the contention of this paper that a sensible vector electrodynamics exists for some special values of meson mass and charge.[19]

In considering the implications of $Z=0$ it is perhaps instructive to clarify the relationship of the field $A(x)$ to the so-called unrenormalized field $A_u(x)=Z^{+1/2}A(x)$. One could perfectly well rewrite the entire theory formally in terms of $A_u(x)$ so that the Z factors disappear from the Lagrangian. One may now require $Z(e^2)=0$ but the important remark is that even if this is the case, unlike (23) and (25), the equations satisfied by Δ_u^{-1} and Γ_u are not homogeneous. Thus two distinct situations may[20] be envisaged:

(A) Field $A_u(x)$ describes the physical situation. The propagator Δ_u has no pole ($Z=0$). Thus there is no stable physical particle and the conventional measurement of meson's electric charge e (using limiting static electric fields) presents conceptual difficulties. Most important of all, there is no dimunition in the divergence of the theory.

(B): Field $A(x)$ describes a stable particle of mass m. In this case the unrenormalized fields and particles have no meaning whatever. The "true" field $A(x)$ may correspond possibly to nonelementary spin-one particles (like the deuteron) and a restriction on mass and charge $Z(e^2,m^2)=0$ is a necessity for the theory to make sense. The role of conditions like $Z=0$ in connection with theories of composite particles has been discussed earlier.[21] It was, of course, not appreciated then, that the same condition would also prove necessary for renormalizability[22] for spins other than 0 and $\frac{1}{2}$.

For interaction of spin-one particles with fermions, it is clear that besides the vanishing of the meson wave-function renormalization constant, ($Z_2=0$) we shall also need $Z_1=0$ where Z_1 renormalizes the vertex part. Thus there must be a functional relationship between the coupling constant and fermion and meson masses.

In reference (20) we envisaged theories with $Z_1=Z_2=Z_3=0$. In a future paper the convergence properties of integrals in such theories are investigated. We conjecture that such field theories have no infinities whatever and that quantum theory of fields is a subject wrongfully, unduly, and much maligned in the past, principally by its friends.

The author is indebted to S. Kamefuchi, G. Feldman, P. T. Matthews, and L. Brown for stimulating discussions.

corresponding condition is $E_m+E_p>4$. Unlike the case of scalar electrodynamics, meson-meson scattering seems to be convergent for spin-one particles.

[18] Since m_0^2 always occurs in the combination Zm_0^2, for $Z=0$, a finite m_0^2 would imply a second relation between e^2 and m^2, i.e.,

$$\int \frac{K^2-m^2}{K^2}G_1(K^2)dK^2=\int G_2(K^2)dK^2,$$

which is unlikely to be true in general. {It certainly does not hold for an electrodynamics of the type suggested by A. Salam and J. C. Ward [Nuovo Cimento **11**, 568 (1959)] where $\partial j_\mu^\pm/\partial x_\mu=0$ and, therefore, $G_2=0$.}

[19] Thus an expansion around $e=0$, is unthinkable. It is perhaps in this sense that there may be some correspondence between the present paper and recent work of T. D. Lee on renormalization of vector-electrodynamics where matrix elements are shown to depend on $e\ln e$. [T. D. Lee, Phys. Rev. **128**, 899 (1962); see also C. N. Yang and T. D. Lee, **128**, 855 (1962)].

[20] I am indebted to Professor G. Feldman for the following simple antithesis between Cases (A) and (B). Since $Z\approx|\langle\text{bare}|\text{true}\rangle|^2$, $Z=0$ means: either (A) $|\text{true}\rangle\equiv0$, $|\text{bare}\rangle\neq0$; no true stable particle exists and there is no "renormalized" field; or (B) $|\text{bare}\rangle\equiv0$; $|\text{true}\rangle\neq0$; there is no "elementary" field $A_u(x)$; however the "true" field $A(x)$ exists and corresponds possibly to a composite particle.

[21] A. Salam, Nuovo Cimento **25**, 224, 1962; 1962 International Conference on High Energy Physics, CERN, p. 686. S. Weinberg *Op. cit.* p. 683. M. J. Vaughan, R. Aaron, and R. D. Amado, Phys. Rev. **124**, 1258 (1961). J. C. Howard, and B. Jouvet, Nuovo Cimento **18**, 466, 1960. R. Acharya, Rochester preprint NYO 10125. S. Weinberg, Phys. Rev. **130**, 776 (1963). I am indebt to Professor Steven Weinberg for stressing to me the virtues of "zero Lagrangians."

[22] In an early paper S. F. Edwards, Phys. Rev. **90**, 282 (1953) did point out that $Z=0$ is a necessary condition for the solution of $\Gamma_1=Z+K[\Gamma_1]$ for the electrodynamics of spin-$\frac{1}{2}$ particles. In his paper, however, the coupled equation for Δ^{-1} was not simultaneously considered so that it is somewhat tricky to compare his results with ours. When dealing with non-gauge-invariant theories where no Ward identity exists, it is, of course, unlikely that for the basic unit $\Gamma_1\Delta_1\sim1/p^\alpha$, α will equal 1, and a more complicated behavior may be expected.